Abstract

High order material point method

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R.P.W.M. Tielen

Technische Universiteit Delft

TU Delft Delft University of Technology

Deltares Enabling Delta Life

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The material point method (MPM) is a meshfree mixed Lagrangian-Eulerian method which utilizes moving Lagrangian material points that store physical properties of a deforming continuum and a fixed Eulerian finite element mesh to solve the equations of motion for individual time steps. MPM proved to be successful in simulating mechanical problems which involve large deformations of history-dependent materials.

The solution on the background grid is found in MPM by a variational formulation. The integrals resulting from this formulation are numerically approximated by using the material points as integration points. However, the quality of this numerical quadrature rule decreases when the material points become unevenly distributed inside the mesh.

It is common practice in MPM to adopt piecewise linear basis functions for approximating the solution of the variational form. A problem arises from the discontinuity of the gradients of these basis functions at element boundaries leading to unphysical oscillations of computed stresses when material points cross element boundaries. Such grid crossing errors significantly affect the quality of the numerical solution and may lead to a lack of spatial convergence.

As a remedy to these problems, a version of the MPM making use of quadratic B-spline basis functions is presented. The C^0-continuity of their gradients eliminates grid crossing errors. Hence, a more accurate reproduction of physical quantities such as stresses and velocity is obtained. Using spline interpolation allows to more accurately approximate integrals, which enables the use of a coarser mesh. This in turn results in lower computational effort.

To improve spatial convergence, the use of a consistent mass matrix instead of a lumped one commonly used with the MPM is suggested to project velocities from material points to the grid more accurately. Explicitly solving the linear system is avoided by using Richardson iteration. Improvements in terms of higher accuracy and rate of convergence are demonstrated for 1D benchmarks involving small and large deformations. In particular a vibrating bar and a column subjected to loading are considered.

This master project has been carried out in the period from October 2015 until July 2016 with support and in collaboration with Deltares, a Dutch research and consulting company that is developing MPM software to simulate geotechnical problems.