

Abstract

A hybrid particle-mesh method for
simulating free surface flows

J. M. Maljaars

Technische Universiteit Delft

Hybrid particle-mesh methods attempt to combine the advantages of Eulerian and Lagrangian methods: Lagrangian particles are used for the advection, whereas a Eulerian background grid is used for computing the particle interactions. Such a hybrid approach is expected to have several benefits when simulating flows involving free surfaces or material interfaces: the particles can be efficiently used to track the free surface, while the background grid can be used to solve the governing Navier-Stokes equations and impose the incompressibility constraint in a convenient manner. The prospects of these hybrid particle-mesh methods for simulating incompressible fluid flows involving a free surface are assessed in this thesis. More specifically, the feasibility of setting-up a numerical wave flume using hybrid particle-mesh methods is investigated.

Four main steps are common to all hybrid particle-mesh methods: particle-to-grid mapping, solving the equations at the background grid, grid-to-particle mapping and particle advection. Due to the large variety of possible model options, it is impossible to speak of *the* hybrid particle-mesh method. Main difference between the various approaches is how the particle-grid interaction is incorporated. Depending on the specific implementation, the hybrid particle-mesh methods can be either regarded to be a Eulerian method augmented with Lagrangian particles, or a Lagrangian method in which the Eulerian background grid only serves as a useful tool for solving the governing equations.

A specific implementation of the four main steps comprising the hybrid particle-mesh methods is presented assigning relatively much importance to the background grid. The particle-to-grid mapping is presented in terms of a weighted least square mapping. In the specific implementation of this least square mapping, the grid nodes are considered to be sample points of the continuum. Following this strategy, an optimal quadrature rule can be used for numerical integration of the variational form arising from the finite element discretization of the governing equations at the background grid.

The specific implementation of advecting the particles in the grid velocity field, combined with the employed, admissible P_0P_1 element, called for an additional mapping of the (divergence free) velocities to a (non-divergence free) continuous velocity field, a step which can at best be regarded as an engineering solution. Assessing different advection schemes revealed the higher order scheme (RK3) to be an economical choice for doing the particle advection.

Various test cases were run, both for single-phase and two-phase problems, showing the advantages and the disadvantages of the developed model. The results obtained for the single-phase problems are in good agreement with analytical results or results obtained with established numerical methods. Employing an admissible element, pathological locking was effectively avoided in the method. Despite these good results, there is clearly some numerical diffusion present in the system. A more fundamental issue was encountered for the advection dominated lid driven cavity test, where the incompressibility constraint is clearly violated at particle level. As a result, unphysical gaps in the particle distribution are observed.

In general, the model results obtained for the two-phase benchmarks are again in good agreement with analytical or experimental results. With an additional mapping of the pressure gradient term, the sharp interface between materials (air-water) is well-maintained over time by the particles, although a residual particle oscillation was observed around the interface. Distinct advantage of the hybrid particle-mesh method is the implementation of kinematic boundary conditions. Since the mesh can be redefined every timestep without additional interpolation steps, the method can deal with large boundary displacements as shown for a solitary wave generated with a moving boundary. Finally, using the hybrid particle-mesh formulation, interfaces of complex topological shape are conveniently tracked by the particles. As such, the method was shown to be able so simulate a breaking wave on a submerged bar up to and including wave breaking.

Based on the thesis, it can be concluded that hybrid particle-mesh methods appear to be an attractive tool for simulating free surface flows and simulating the nearshore propagation of waves. Nevertheless, many fundamental questions remain unanswered when considering hybrid methods. It is to be remarked that all these questions can be basically reduced to the question how to interpret the interaction between particles and grid. Future work should therefore primarily focus on this issue.