Modelling of pile installation using the material point method (MPM)

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ABSTRACT: During the installation of displacement piles the soil around the pile is heavily distorted, which involves large shear deformation especially along the pile shaft and compression especially around the pile tip. This leads to a change of the stress and density state and therefore also to the soil behaviour in the adjacent area around the pile and has a significant influence on the pile bearing capacity. In general, numerical simulations of displacement piles with the classical Finite Element Method (FEM) cannot account for installation effects due to significant mesh distortion. Using the Material Point Method (MPM), however, shortcomings due to the occurring large deformations can be circumvented. In this method (Lagrangian) material points carrying the state parameters move through a (Eulerian) background mesh at which the equations of motion are solved. In the current paper MPM simulations of the installation of a displacement pile in Baskarp sand using a Mohr-Coulomb model are presented. The numerical results are compared with centrifuge tests at 40g. The comparison indicates that MPM is well suited to simulate the installation process of displacement piles as well as subsequent load tests in granular soil.

1 INTRODUCTION

During the installation of displacement piles the soil around the pile is heavily distorted, as shown in recent laboratory experiments by e.g. Dijkstra (2009). This leads not only to a change of the density around the pile, but also influences the stress state in the soil. Moreover, a higher pile bearing capacity can be expected if the soil is compacted in combination with a high confining stress around the pile (e.g. Lehane & White 2005).

Such installation effects are included in empirical or analytical design methods used in engineering practice. However, for more complex geotechnical constructions, numerical analyses using the finite element method (FEM) are more common. Current numerical methods for the determination of the pile bearing capacity, however, cannot properly account for the specific behaviour during pile installation due to the occurrence of large deformations and therefore heavy mesh distortion (Dijkstra 2009). Moreover, the interaction between pile and soil as well as the change of soil state and properties during the installation process are often not taken into account. However, as these complex effects are considered to be essential to make accurate and reliable predictions of the pile bearing capacity, the influence of the installation process on the adjacent soil and eventually structures needs to be taken into account.

The numerical methods need to be capable to handle effects of large deformations occurring during pile installation. At present, the classical finite element method (FEM) is not able to provide reliable solutions due to mesh distortions which cannot be properly captured using an (updated) Langrangian scheme. In contrast to the purely Lagrangian and updated Lagrangian methods, the Eulerian and Arbitrary Lagrangian-Eulerian (ALE) schemes (e.g. Gadala 2004) allow for uncoupling of mesh and material and consequently for an independent movement of the material with respect to the mesh. It helps to overcome the issues of mesh distortion. However, in the case of remeshing, the mapping of state variables allocated to the material introduces additional errors into the calculation. This has led to the development of meshless methods such as the smoothed particle hydrodynamics (SPH) method (Monaghan 1988) and mesh-based particle methods such as the Material Point Method (MPM), e.g. Coetzee et al. (2005), Beuth (2012), Jassim (2013), Bandara (2013). MPM can be regarded as FEM formulated in an ALE description of motion. It uses two kinds of space discretization: first the computational mesh and second the collection of material points which move through the fixed mesh. The advantage
of MPM is that the state variables are traced automatically by the material points, and are therefore carried independently of the computational mesh. Therefore MPM is well suited to model problems in which large deformations occur.

2 BACKGROUND INFORMATION

2.1 Basic concepts of MPM

A general description of the Material Point Method is provided in e.g. Beuth (2012), Jassim (2013) or Bandara (2013). The governing equation of MPM is given in Eq. 1, which is identical for the explicit formulation of FEM and MPM,

\[ \mathbf{M} \mathbf{u} = \mathbf{F}^{\text{ext}} - \mathbf{F}^{\text{int}} , \]

where \( \mathbf{M} \) is the (lumped) mass matrix, \( \mathbf{u} \) is the vector of nodal displacement, and \( \mathbf{F}^{\text{ext}} \) and \( \mathbf{F}^{\text{int}} \) are the vectors of external and internal nodal forces, respectively. However, in MPM the mass matrix varies with time since the material points move through the mesh. In other words, the total number of degrees of freedom of the system can vary.

The calculation process of MPM which is shown in Figure 1 can be subdivided into three steps: the initialization phase, the Lagrangian phase and the convective phase. Initially, data defining the given problem are assigned to the material points at the current positions. At the beginning of the calculation phase, the information carried by the material points is transferred to the nodes of the computational mesh. At the mesh, Eq. (1) is solved in an updated Lagrangian frame, forming the Lagrangian phase of the calculation. At the end of this phase, the properties of the material points are updated (position, velocity, strain and stress). During the convective phase of calculation, the mesh is reset to its initial configuration, whilst the material points keep their position and the properties are unchanged compared to the end of the Lagrangian phase.

2.2 Description of the centrifuge test

For the validation of the MPM simulations of pile installation and load tests, results of centrifuge tests carried out at Deltares are available. The centrifuge experiments performed at Deltares are described in detail in Huy (2008) and Holscher et al. (2012). The tests were carried out in a 0.6 m diameter and 0.79 m high steel container filled with sand (height 0.46 m). The steel pile has a diameter of \( D = 11.3 \) mm and a flat pile tip. During preparation at 1 g the pile was initially embedded at \( 10D \) below the sand surface. The installation of the model pile started after the centrifuge had been accelerated to a level of 40 g. During this installation phase the pile was pushed into the soil with a velocity of \( 10 \) mm/min (0.167 mm/s) up to a final depth of \( 20D \) (10D pre-embedded and 10D penetration). To determine the bearing capacity of the pile after installation a static load test (SLT) was performed with a velocity of 0.00167 mm/s and an additional displacement of 0.1D.

3 NUMERICAL MODEL

The Material Point Method (see Section 2.1) is used to simulate the pile installation experiment performed in the geotechnical centrifuge (see Section 2.2). In order to enable the comparison of results, the numerical simulations are carried out at the model scale in this study, i.e. the initial stresses in the model are created at 1 g and the spinning up of the centrifuge is included in the numerical analysis. In the following, the main modelling techniques are explained.

3.1 Geometry and mesh discretization

Axisymmetric conditions are assumed as only a soil wedge of 20 degrees is modelled. The pile problem lends itself for axisymmetric conditions and by that computational time can be reduced compared to a full 3D model. The mesh discretisation with 26,826 tetrahedral elements and 152020 material points is shown in Figure 2. Note that initially also inactive elements

Figure 1. The MPM calculation process subdivided into Lagrangian phase (left) and Eulerian phase (right).

Figure 2. Mesh discretization used for the numerical analysis with MPM.
are present in the area above the soil surface. These elements may be activated during the calculation process as soon as material points are entering. The right side boundary is at a distance of \(26D\) from the pile centre which is identical to the size of the sample container in the centrifuge experiment. The bottom boundary is fixed in all coordinate directions. The right side boundary is fixed only in normal direction and free in the other directions.

### 3.2 Moving mesh concept

To increase the accuracy, the mesh is finer around the pile tip. As the pile is moving through the mesh during installation one would need a fine mesh along the complete penetration depth in the standard formulation of MPM. In order to avoid this the moving mesh concept was developed. The computational domain is divided into two zones: a moving mesh and a compression mesh. The moving mesh is attached to the pile such that it moves with the same displacement as the pile. During the computation, the elements of this zone keep the same shape while the elements in the compressed mesh zone are compressed. The details of the moving mesh concept are described in Jassim (2013). By using this procedure, the part with the fine mesh will always remain around the pile tip.

### 3.3 Pile tip geometry

For all computations the diameter of the model pile is 11.3 mm with a tip angle of 68 degree and a tip length of 12.8 mm. The transition from the shaft to the tip is curved. This smooth shape of the pile tip avoids numerical difficulties due to locking.

### 3.4 Pile soil interaction

Modelling pile installation in soil involves the problem of soil-structure interaction. The material point method is naturally capable of handling non-slip contact between different bodies without any additional algorithm, and inter-penetration between different bodies cannot occur. However, when pile and soil come into contact in most cases frictional sliding occurs at the contact surface. To model such sliding and interaction a contact algorithm that allows relative motion at the contact surface is required. The detail of the frictional contact algorithm which used in this study is described in Jassim (2013). The friction angle of sand against polished steel is about 11° as suggested by Murray and Geddes (1987). Therefore a coefficient of friction \(\mu = \tan \phi = 0.176\) is used for all simulations.

### 3.5 Installation process

The pile is considered as a rigid body as no wave propagation in the pile is observed due to the relatively low installation velocity. The pile is initially embedded at \(10D\) below the sand surface, identical to the centrifuge experiments. The initial stresses in the soil are determined by a K0-procedure. During the installation process, the pile was pushed into the soil by \(10D\) with a velocity of \(0.02\) m/s. The simulation of the installation is performed at a gravity level of \(40\) g which is identical to the conditions in the centrifuge test. After the simulation of the pile installation a relaxation phase follows. The relaxation phase was modelled by applying a prescribed velocity on the pile head in order to pull the pile upwards to unload it. Finally the static load test (SLT) is performed with a velocity of \(0.002\) m/s. The velocity used in the numerical analyses is higher than the value used in the centrifuge test in order to increase calculation performance. In a parametric study it has been confirmed that the increased velocity is still acceptable to produce reliable results.

### 4 SOIL BEHAVIOUR

During the installation process the soil around the pile tip is exposed to (very) high stress levels. The maximum stress at the pile tip towards the end of the installation process can be estimated to be around 8 MPa for medium dense sand and 5 MPa for loose sand respectively (Huy 2008). Therefore the constitutive model used for the numerical simulations needs to be able to account for changing soil response during a wide range of stress levels. Luong & Touati (1983) show for sand in triaxial testing conditions that for increasing cell pressure from 0.5 MPa to 30 MPa the strength and dilatancy are both decreasing significantly. For cell pressures in the range between 6 MPa and 30 MPa zero dilation is observed. Bolton (1986) discussed the stress dependency of the friction and dilation angle for sands, and derived the following relations:

\[ \varphi'_{\text{max}} - \varphi'_{\text{crit}} = 3I_R, \]

in which \(\varphi'_{\text{max}}\) and \(\varphi'_{\text{crit}}\) are the maximum and critical friction angle respectively, and the relative dilatancy index \(I_R\) is defined as,

\[ I_R = I_D(Q - \ln(p')) - R, \]

in which \(I_D\) is the relative density of sand, \(p'\) is the applied mean effective stress level. \(Q, R\) are relative dilatancy indices for which Bolton suggested the values \(Q = 10\) and \(R = 1\) for quartz sand. The maximum dilation rate at failure state is defined as,

\[ \frac{d\varepsilon_{\text{vert}}}{d\varepsilon_{\text{rot}}} \bigg|_{\text{max}} = 0.3I_R. \]

Furthermore, Schanz & Vermeer (1996) showed that the dilation angle can be defined by,

\[ \sin \psi = -\frac{\frac{d\varepsilon_{\text{rot}}}{d\varepsilon_{\text{vert}}}}{2 - \frac{d\varepsilon_{\text{rot}}}{d\varepsilon_{\text{vert}}}}. \]
Table 1. Calculated maximum friction and dilation angle based on relations by Bolton and Schanz for medium dense and loose sand at low and high stress levels.

<table>
<thead>
<tr>
<th>Sand</th>
<th>ID</th>
<th>p [kPa]</th>
<th>IR</th>
<th>(\phi_{crit})</th>
<th>(\phi_{max})</th>
<th>(\psi_{max})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Med. dense</td>
<td>0.54</td>
<td>100</td>
<td>1.91</td>
<td>31</td>
<td>36.74</td>
<td>12.89</td>
</tr>
<tr>
<td>Med. dense</td>
<td>0.54</td>
<td>8000</td>
<td>-0.45</td>
<td>31</td>
<td>29.64</td>
<td>-4.18</td>
</tr>
<tr>
<td>Loose</td>
<td>0.36</td>
<td>100</td>
<td>0.94</td>
<td>31</td>
<td>33.83</td>
<td>7.11</td>
</tr>
<tr>
<td>Loose</td>
<td>0.36</td>
<td>5000</td>
<td>-0.47</td>
<td>31</td>
<td>29.60</td>
<td>-4.31</td>
</tr>
</tbody>
</table>

Table 2. Mohr-Coulomb material parameters for medium dense and loose sand used in the MPM simulations.

<table>
<thead>
<tr>
<th>Sand</th>
<th>ID</th>
<th>E [kPa]</th>
<th>(\phi_{max})</th>
<th>(\psi_{max})</th>
<th>c [kPa]</th>
<th>(\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Med. dense</td>
<td>0.54</td>
<td>40000</td>
<td>30°</td>
<td>0°</td>
<td>1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Loose</td>
<td>0.36</td>
<td>22000</td>
<td>30°</td>
<td>0°</td>
<td>1.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Combining Eqs. (4) and (5), the dilation angle can be expressed in the form,

\[
\sin \psi = \frac{0.3I_R}{2 + 0.3I_R}.
\]  

Consequently, for Baskarp sand at stress level of 8 MPa (for medium dense sand) and 5 MPa (for loose sand), the maximum friction angle is around 30° and slightly negative dilation (around \(-4^\circ\)) as shown in Table 1. Therefore, it can be concluded that the relations derived by Bolton and Schanz are in good agreement with the triaxial test results of Luong and Touati (1983).

In this paper the elasto-plastic Mohr Coulomb model is used to model the behaviour of the sand. In order to avoid an overestimation of the pile head force in the simulation of the centrifuge test due to the dependency of friction and dilation angle on the stress level, it is suggested to use the low values for the friction dilation angle as discussed above. The material parameters for the Mohr Coulomb model are summarised in Table 2. All simulations are carried out in dry conditions.

5 RESULTS

5.1 Load-displacement curve during installation

In order to compare the numerical results with the centrifuge tests the evolution of the axial forces on the pile head is investigated. The simulations show a good overall fit of the load-displacement curve in both cases of loose and medium dense sand (Figure 3). It is observed that in the early penetration stage (low depths) the numerical results show a softer behaviour compared to the stiffer response in the experiments. This may be explained as being a consequence of the assumption of low mobilised friction angle and zero dilation angle which remain constant during the whole installation process.

Figure 3 also shows the increase of shaft resistance with depth during installation in the MPM simulations, which is generally in good agreement with the test results of Dijkstra (2009). At the end of the installation process, the shaft resistance is about 25% and 33% of the total pile head force for medium dense sand and loose sand respectively.

5.2 Load-displacement curve during SLT

With respect to the static load test (SLT) the MPM simulations show good agreement of the pile bearing capacity with the centrifuge tests as shown in Figure 4. The pile head load in the MPM simulations is slightly lower than one of centrifuge tests which might be a consequence of the assumption of low friction and dilation angle during the SLT. It can also been seen that, without installation effects, the calculated bearing capacity of the pile is significantly lower than the result of the test. Therefore, it can be concluded that the installation effects need to be taken into account when simulating the SLT.

5.3 Stress state after installation

In Figure 5 the distribution of radial stresses around the pile tip is shown for both loose and medium dense sand after installation of 10\(D\). In both cases the radial
Figure 4. Load-displacement curve of the pile head during SLT for medium dense (top) and loose sand (bottom). Comparison of centifuge test results and MPM simulations with and without installation effects.

Figure 5. Radial stresses after $10D$ penetration for medium dense (top) and loose (bottom) sand.

Figure 6. Radial stresses after $10D$ penetration at horizontal cross section $A - A'$ for medium dense and loose sand.

Figure 7. Radial stresses after $10D$ penetration at a vertical cross section $B - B'$ close to the pile shaft for medium dense and loose sand.

Stresses increase during installation and exceed the values at $K_0$-state. The radial stresses near the pile tip in medium dense sand are nearly double those in loose sand, which can also be seen in Figure 6. In the figure, radial stresses are shown for a horizontal cross section $A - A'$ after installation of $10D$. For a distance from the pile centre larger than $10D$, no stress changes can be observed anymore for both cases.

Figure 7 shows the radial stresses in a vertical cross section close to the pile shaft. Along the pile shaft a significantly higher horizontal stress compared to the $K_0$ stress state is observed even after the installation process has finished. At the pile tip a peak of horizontal stress can be seen. The peak value for medium dense sand is about 2300 kPa which is nearly 13 times the initial stress. For the loose sand the stresses are lower.
and reach a peak value of around 1500 kPa, which is around 8 times the initial stress. Below the tip the horizontal stresses drop down to a value below the $K_0$ state for both cases. The observed change of radial stresses along a vertical cross section near the pile shaft is in good agreement with Mahutka et al. (2006).

5.4 Density change after installation

Although in the Mohr-Coulomb model the change of density is not affecting the resulting stresses, through the change of volumetric strain the zones of dilation and compaction can be identified as shown in Figure 8. For medium dense sand a thin dilative zone along the pile shaft can be observed, while in loose sand only densification can be seen. The dilative zone may be explained due to the high shear force between the pile and the soil. Hence, close to the pile the compaction of the soil is superimposed by the shearing process while in a greater distance from the pile the compaction is predominant (Mahutka et al. 2006).

6 CONCLUSIONS

The MPM simulations show good agreement with centrifuge test results for both installation process as well as the static load test (SLT). Therefore, it can be concluded that the MPM code used in this study is well suited to model the large deformations occurring during the pile installation process.

In order to model successfully the centrifuge test, it is very important to account for the dependency of friction and dilation angle on the stress level. Therefore, the use of relations derived by Bolton (1986) and Schanz & Vermeer (1996) helps to avoid an overestimation of the pile head force in the simulations.

The numerical analyses of pile installation show significant differences of the soil stresses around the pile after installation. At the pile tip, very high lateral stresses are observed due to the soil is pushed aside by the pile causes a stronger interlocking of the soil grains. As a consequence of such changed soil state after installation a significantly higher pile bearing capacity is observed compared to the one without installation effects. Therefore the importance of including installation effects in the determination of the pile bearing capacity is demonstrated.

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